


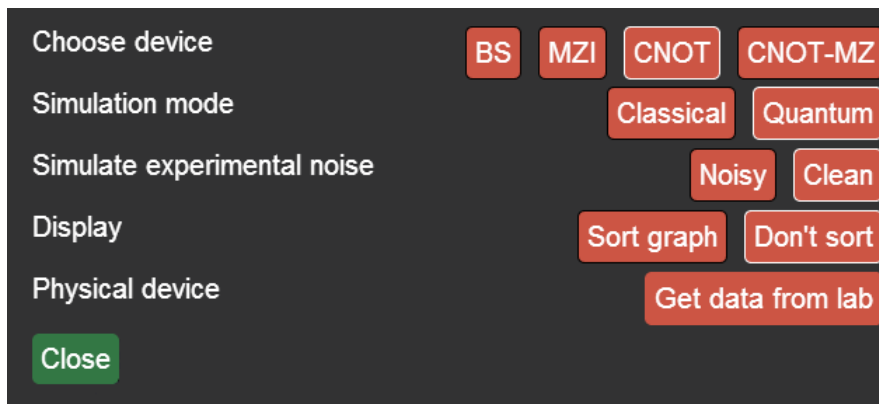
# How to use the simulator

## Overview

The application allows for the exploration of four quantum circuits in all. Each simulator grants the user a large amount of freedom in experiments they wish to conduct. Each phase gate is tunable to a  $0.01\pi$  radians precision, and photons can be injected to any combination of paths. As a default, quantum interference is visible, as this is the regime in which we wish to work for experiments, but the user has the option to turn this off completely in order to examine the way purely classical interference affects results. Experimental noise can be added in order to make simulations more realistic. Outputs are available both in the form of a histogram, with heights representing probabilities of photons finishing in a certain path / path combinations, or as numerical data.

## Settings Menu

Clicking on the  icon in the top-right corner takes you to the settings menu:



Here, you have control over the general behaviour of the simulator in the following areas:

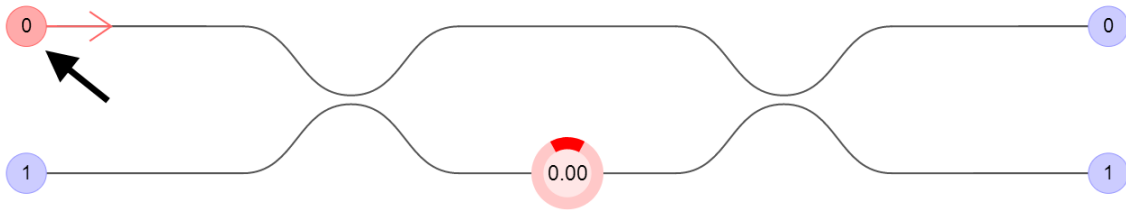
- **Choose device:** Click on either BS (Beam Splitter), CNOT (Controlled NOT gate), MZI (Mach Zehnder Interferometer), CNOT-MZ (the chip that we have set up in the lab in Bristol). Both BS and CNOT are more demonstrational tools than for performing experiments, but there is still some interesting physics going on in both. Selecting BS only lets you choose a maximum of 2 paths to inject photons, while CNOT allows for 4 input paths. Choosing MZI will give you access to slightly more sophisticated experiments, with a maximum of 2 photons in 2 paths and one phase shifter to play around with, whereas choosing CNOT-MZ gives a maximum of 4 photons in 6 paths, with 8 variable phase shifters - for the most complex experiment designs.
- **Simulation mode:** Clicking “Classical” will ignore all quantum interference effects. This is equivalent to performing the experiment with a time delay between the injected photons, or in any other way making the photons distinguishable from one another. The “Quantum” setting gives the ideal interference regime for performing most of the experiments we want to carry out, and is especially important for the correct function of the CNOT gate.
- **Simulate experimental noise:** The number of detected photons follows a Poissonian distribution, and as such there will be an amount of noise visible in experimentally obtained photon counts. Clicking “Noisy” will apply a model of this effect based on  $\sim 1000$  detected photons, and displayed numbers will be in the form of **number of photons detected**. Choosing “Clean” means that an idealised experiment will be performed with just one photon injected in to each selected mode. In this case, the results displayed correspond to a **probability** that the detector(s) will click in the given path(s).
- **Display:** If you would prefer to have the histogram arranged in left-to-right descending order of probabilities/counts, click “Sort graph”. Otherwise, “Don’t sort” will order in terms of the path labelling.
- **Physical Device:** Coming soon.

Once you are happy with the settings, press “Close” to start designing your experiment.

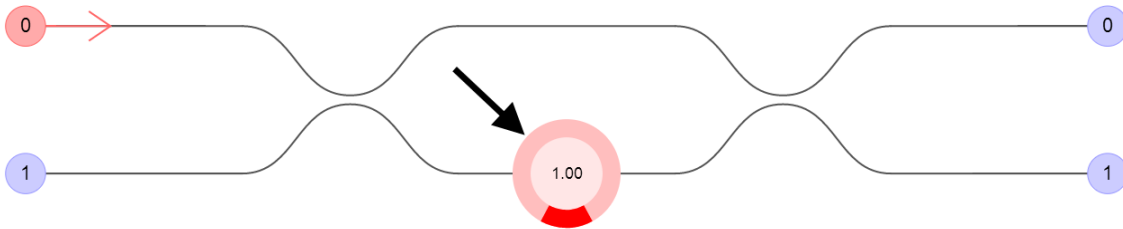
# General Operation

The way in which you operate the simulator for all devices is essentially the same, with the following steps:

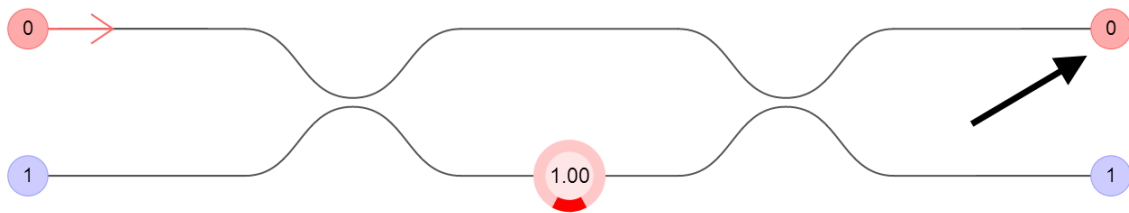
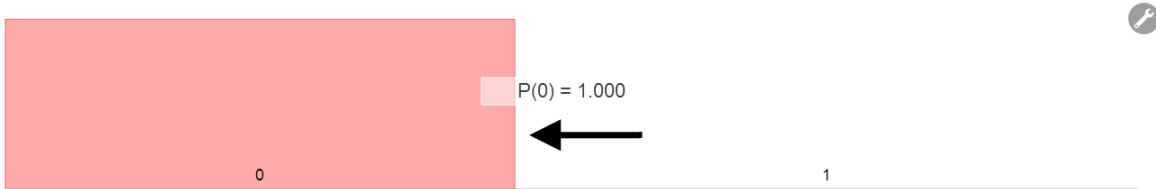
1. Decide which paths in which you want to inject photons, and click on the relevant numbers on the **left hand side** of the device. If you do anything wrong here, just click a number again to stop injecting photons in that path.



2. **Skip this for the BS and CNOT devices:** Decide any values of phase you wish to apply, and set the value(s) on the relevant phase shifter(s). You do this by clicking on the phase shifter, moving your mouse round the circle until you reach the correct number, and clicking again to confirm. Note that the units for each phase are  $\pi \cdot$  radians.



3. Choose output paths to measure photons in (or display probabilities for, if noise is off) by clicking the relevant numbers on the **right hand side** or the relevant sections of the **histogram** above the chip.



4. Interpret the data.

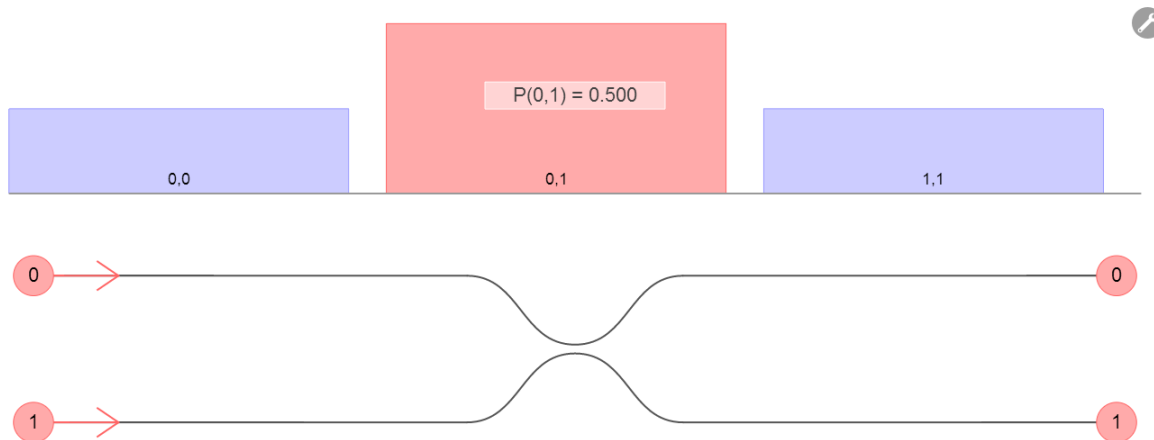
See the final paragraph of the CNOT section of the background information document in order to find out more information on how to produce meaningful results with the CNOT-MZ device.

## Examples

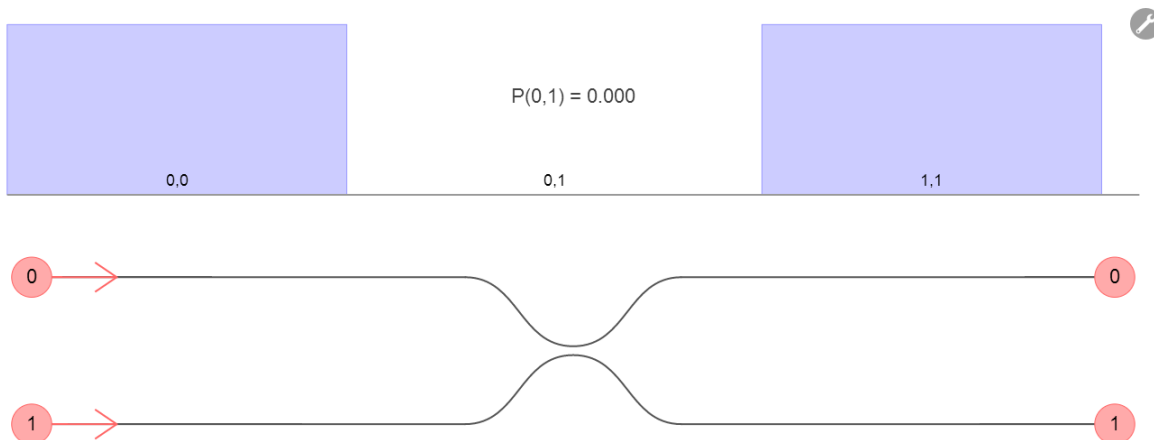
### BS (Beam Splitter)

#### Classical vs. Quantum Interference

Here we will perform a simple experiment in order to see the difference between classical and quantum interference. Firstly, we set up a BS with the simulation mode set to “classical”. If we inject photons in to both input paths, we see the following:



Half of the time, each detector clicks once (a detector clicking means that a photon has been detected in that path) . The other half of the time, one of the detectors clicks twice (split equally between this happening at detector 1 and detector 0). Now, let’s turn quantum interference on and see what happens:



Now, the case of each detector clicking once **never** happens. Both photons always end up in the same path, causing one of the detectors to click twice. This is a purely quantum mechanical effect - it can’t be explained with classical physics. See the quantum interference section of the background information document for more on this effect.

## CNOT

### Postselecting and Guessing the Truth Table

An important concept in the field of processing is that of postselection. Postselection is the act of restricting **outcomes** of a process or experiment, based on certain conditions being satisfied. In order to use our CNOT circuit as a logical CNOT gate, we must apply certain restrictions. We define 2 qubits (again, see the background information): the logical 0 and 1 states of qubit A are a photon in paths 1 and 2 respectively. Similarly, the logical 0 and 1 states of qubit B are

defined by a photon being present in paths 3 and 4 respectively. At the end of the chip, where the detectors are, we only consider the cases in which one photon is detected in one of qubit A's paths *and* one photon is detected in one of qubit B's paths. The other cases have not been defined, so we simply discard them. Now, our CNOT gate provides the sorts of outcomes that we are after with probability 1/9, and if we really are ignoring the times that it doesn't do this, we had better make the probabilities of our relevant outcomes add up to 1. This is done by simply multiplying the probability of each of these outcomes by 9 - we call this **renormalisation**.

Now, let's apply this in order to guess the truth table for our CNOT gate. Call qubits A and B the control and target qubit respectively, and find out the "After" section of the truth table below by injecting photons in to the paths mentioned above (one control and one target at a time!), and seeing where they end up, taking in to account postselection.

*Hint 1: you shouldn't get the same truth table as in the background information.*

*Hint 2: see the Appendix for where to inject photons for each line of the truth table*

Before		After	
Control	Target	Control	Target
0	0		
0	1		
1	0		
1	1		

Note: This is not quite the real truth table for our CNOT gate. Using this method, it is only possible to obtain each result up to a complex quantum **phase factor**. Unfortunately, these aren't measurable, so more sophisticated techniques (such as quantum state tomography) are required.

## MZI (Mach Zehnder Interferometer)

### Basic Quantum Metrology: Super-resolved Interference Fringe

First of all, we'll input photons in just one path – let's say path 0. We're going to keep track of the  $P(1)$  value, so click on output number 1. Gradually turn the phase shifter from 0 to 2, plotting  $P(1)$  against phase angle for a few different values (make sure you get data for at least phase values of 0, 0.5, 1, 1.5 and 2). How many peaks do we see in this interval?

Now, follow the same procedure with photons injected in path 1 in addition to path 0. This time, we are interested in value of  $P(0, 1)$ . Plot  $P(0, 1)$  against phase shifter value again. How many peaks do we see this time?

In the two photon case, we have what is known as a super-resolved interference fringe. We can use this to enhance the precision of measurements beyond what is possible with a non-quantum measurement scheme. Measurements have already been performed using techniques based on this principle, including the concentration of protein in a solution. For more information on this, see: Crespi, A. et al. Measuring protein concentration with entangled photons. Appl. Phys. Lett. 100, 233704 (2012).

## CNOT-MZ

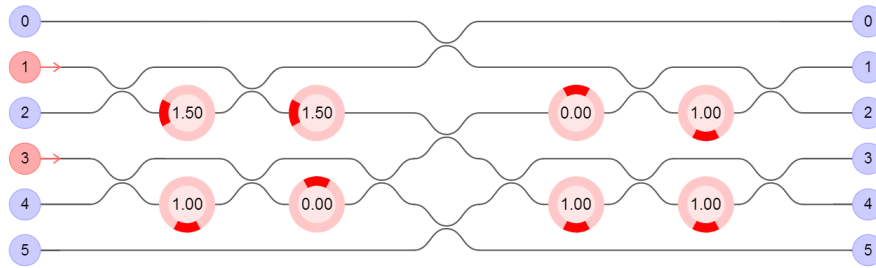
An important thing to note when using this chip, is that arbitrary state generation is only possible when the input is restricted to simultaneous injection in paths 1 and 3 (corresponding to having the initial state  $|00\rangle$ ).

### Preparing States, Measuring in Different Bases and Violating a CHSH Inequality (Advanced)

In order to understand what is going on here, it is handy to have a bit of prior knowledge of basic quantum mechanics e.g Dirac notation, measurement operators etc.

Firstly, for simplicity, we set our experimental noise to "Clean". We prepare a Singlet state,  $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ , by applying the photon input and phase shifter settings demonstrated in the above image. Restricting ourselves to the two qubit logical basis, as mentioned for the CNOT gate, we see that there is zero probability of finding photons in the  $|00\rangle$  or  $|11\rangle$  states, which is what we expect. What is important here, is that only the first four phase shifters and the CNOT gate were used to prepare this state.

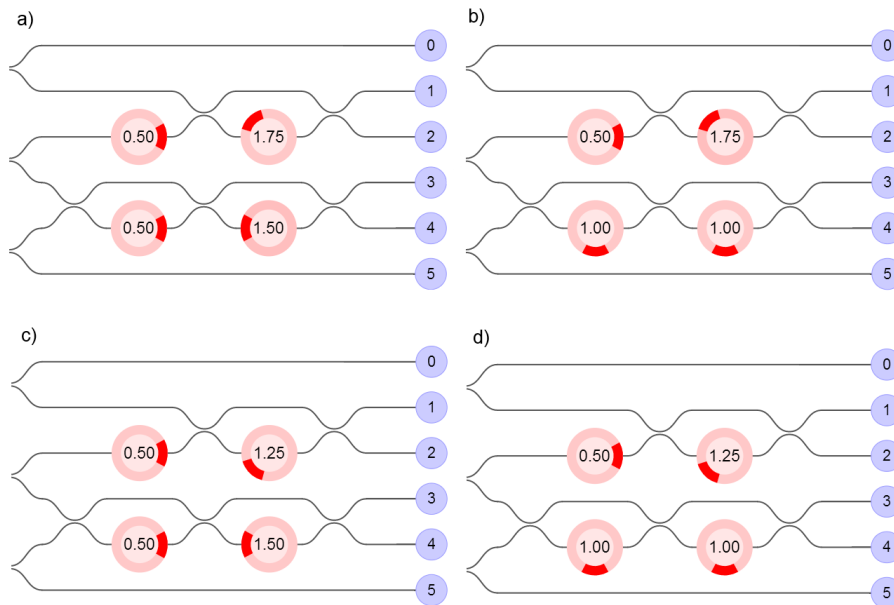
Now for the real part of the experiment - taking measurements. We imagine that we have two experimentalists: Alice, who performs measurements on qubit A (paths 1 and 2), and Bob, who performs measurements on qubit B (paths 3 and 4). Alice and Bob are going to measure their qubits in two different **bases** each, and come together to compare their



results afterwards. Usually, we measure in the  $\hat{Z}$  basis, meaning that we “ask” a photon which of the two paths it is in, but by changing the settings of the final four phase shifters, we can ask more complicated questions. The combination of measurement bases that Alice and Bob measure in are the following:

- a) Alice measures in  $\frac{1}{\sqrt{2}}(\hat{Y} - \hat{Z})$  (call this  $A$ ) and Bob measures in  $\hat{Y}$  (call this  $B$ )
- b) Alice measures in  $\frac{1}{\sqrt{2}}(\hat{Y} - \hat{Z})$  and Bob measures in  $\hat{Z}$  (call this  $B'$ )
- c) Alice measures in  $\frac{1}{\sqrt{2}}(\hat{Y} + \hat{Z})$  (call this  $A'$ ) and Bob measures in  $\hat{Y}$
- d) Alice measures in  $\frac{1}{\sqrt{2}}(\hat{Y} + \hat{Z})$  and Bob measures in  $\hat{Z}$

So in summary, the measurement combinations are:  $(A, B)$ ,  $(A, B')$ ,  $(A', B)$  and  $(A', B')$ . These correspond to us setting the phase shifters as:



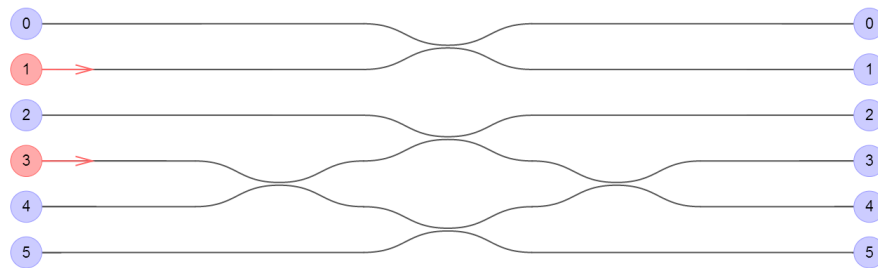
Now, we compare the measurements of Alice and Bob in a very specific way. For each case we calculate how correlated the results are in the following way: Say we are looking at situation a), we define  $E(A, B) = 9[P(1, 3) + P(2, 4) - P(1, 4) - P(2, 3)]$  where each of the  $P(i, j)$  are read off from the histogram obtained from using the settings in a) and the number 9 appears due to renormalisation. Calculate  $E(A, B)$ ,  $E(A, B')$ ,  $E(A', B)$  and  $E(A', B')$ . *Hint: They should all be the same, up to a minus sign.* Now calculate  $E(A, B) + E(A, B') + E(A', B) - E(A', B')$ . Is it smaller, or larger than 2?

The number 2 is significant here. It represents the the largest number that you can possibly get for a non-quantum version of this experiment, and provides a bound for **local realism** – the idea that two objects far away from each other in space are independent of each other, and that externally influencing one can have no direct influence on the other. This experiment shows that quantum mechanics, and therefore reality, is inherently **nonlocal** – something which deeply troubled Einstein.

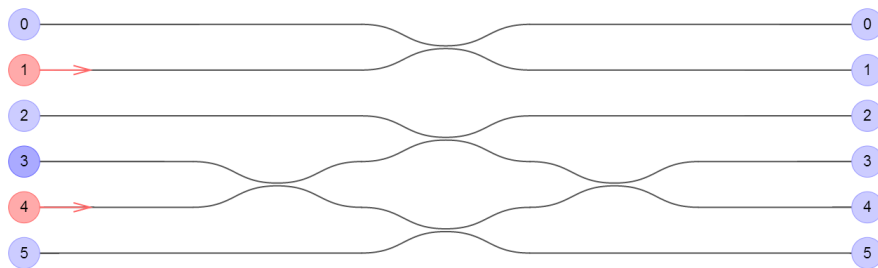
# Appendix

## Inputs for CNOT exercise

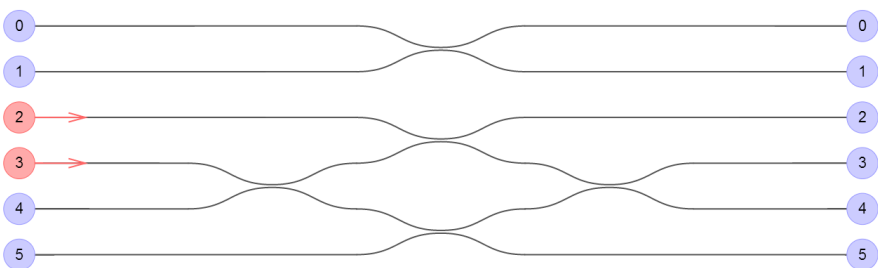
Control 0 and target 0:



Control 0 and target 1:



Control 1 and target 0:



Control 1 and target 1:

